

TABLE OF CONTENTS

ABOUT THE AUTHOR	10
LIST OF GRAPHICAL NOTATIONS AND SYMBOLS.....	12
PREFACE.....	13
I.1. WHO WILL GAIN, AND WHAT, FROM THIS BOOK	13
I.4. ORGANIZATION OF THE BOOK	16
I.5. ACKNOWLEDGEMENTS.....	19
CHAPTER 1.....	20
1. INTRODUCING INTEREST RATE.....	20
IRR EQUATION.....	20
1.1. INTEREST RATE	21
1.1.1. Definition of interest rate	21
1.1.2. The meaning of negative power.....	27
1.1.3. Problems and Exercises:	28
1.2. INTRODUCING COMPOUNDING.....	30
1.2.1. Problems and Exercises:	33
1.3. DOMAINS OF INTEREST RATE AND PERIOD LENGTH	35
1.3.1. Specifics of negative interest rate	35
1.3.2. Can we use a real power in compounding formula?36	
1.3.3. The meaning of compounding when the period length is not an integer value.....	39
1.3.4. Problems and Exercises:	40
1.4. COMPUTING INTEREST RATE FOR DIFFERENT PERIOD LENGTHS.....	42
1.4.1. Relationship between the period length and interest42 rate	42
1.4.2. Computing interest rate for shorter or longer periods. Nominal and effective interest rates.....	44
1.4.3. Mathematical foundations of interest rate calculations	46
1.4.4. Computing interest rates. Numerical examples	49
1.4.5. Problems and Exercises:	52
1.5. CONTINUOUS COMPOUNDING.....	54
1.5.1. Continuous compounding. Definitions	55
1.5.2. Continuous compounding versus discrete compounding. Numerical example	59
1.5.3. Smooth exponential compounding function versus 62 the piecewise linear function	62
1.5.4. Problems and Exercises:	69
1.6. IRR EQUATION. AN INHERENT RELATIONSHIP OF COMPOUNDING OPERATION AND CASH FLOWS	72
1.6.1. How IRR equation accounts for cash transactions. 72 inside the period	72

1.6.2. Deriving simple form of IRR equation	74
1.6.3. Non-compounding scenario.....	78
1.6.4. Problems and Exercises:	79
1.7. DERIVING A GENERAL FORM OF IRR EQUATION	81
1.7.1. IRR Equation with discrete compounding.....	82
1.7.2. IRR equation with continuous compounding.....	84
1.7.3. Getting the “look and feel” of IRR equation.....	86
1.7.4. Problems and Exercises:	91
CHAPTER 2.....	94
ANNUITIES.....	94
2.1. DERIVING AN ORDINARY ANNUITY FROM THE IRR EQUATION	95
2.1.1. Annuity definition and specifics	95
2.1.3. Computing interest rate in case of annuities	100
2.1.5. Annuity’s present value	104
2.1.6. Problems and Exercises:	107
2.2. ANNUITIES AND WITHDRAWALS. PURCHASING AN ANNUITY.....	109
2.2.1. Accumulating required amount.....	111
2.2.2. Finding the number of annuity payments.....	113
2.2.3. Price of an annuity to receive regular payments .	115
2.2.4. Problems and Exercises:	116
2.3. DISCUSSION OF ANNUITY’S FEATURES	119
2.3.1. Annuity with a negative interest rate.....	119
Numerical example	119
2.4. ANNUITY DUE	125
2.4.1. Definition of annuity due	125
2.4.2. Deriving a general formula for an annuity due....	127
2.4.3. Finding regular payment	130
2.4.4. Finding initial payment	131
2.4.5. Determining the number of payments	132
2.4.6. Calculating the present value	133
2.4.7. Notes with regard to relationship between	135
annuities and IRR equation.....	135
2.4.8. Problems and Exercises:	136
2.5. ANNUITIES WITH CONTINUOUS COMPOUNDING.....	139
2.5.1. Ordinary annuity.....	139
2.5.2. Finding a regular payment.....	142
2.5.3. Calculating a lump sum payment	143
2.5.4. Finding the number of payments	145
2.5.5. Computing unknown interest rate	146
2.5.6. Finding the present value.....	147
2.1.7. Problems and Exercises:	148
2.6. RELATIONSHIP BETWEEN INTEREST RATES FOR PERIODS WITH DIFFERENT LENGTHS	151
2.6.1. Computing interest rate for an arbitrary period length	157
CHAPTER 3.....	160
MORTGAGES. DERIVING MORTGAGE FORMULAS FROM	

IRR EQUATION.....	160
3.1. DEFINING MORTGAGE IN MATHEMATICAL TERMS.....	162
3.1.1. Deriving a mortgage formula for remaining balance.....	162
3.1.2. Finding payment amount	165
3.1.3. Calculating number of periods	166
3.1.4. How much can we afford?	168
3.2. NUMERICAL EXAMPLES, GRAPHS AND DISCUSSION	170
3.2.1. Interest and principal amounts	170
3.2.2. Influence of payment frequency	173
3.2.3. Weekly and monthly payments versus annual payments.....	176
3.2.4. Problems and Exercises:	179
3.3. CHOOSING THE RIGHT METHOD TO CALCULATE AN INTEREST RATE FOR SHORTER OR LONGER PERIODS	182
3.3.1. Problems and Exercises:	185
3.4. GENERALIZATION OF MORTGAGE EQUATIONS	187
3.4.1. Proportionality between the period payment and principal amount	187
3.4.2. Relationship between the interest rate and period length	188
3.4.3. Problems and Exercises:	191
3.5. MORTGAGES WITH CONTINUOUS COMPOUNDING	193
3.5.1. Mortgage equations for continuous compounding	195
3.5.2. Finding the remaining balance	196
3.5.3. Calculating payment amount	197
3.5.4. Finding a number of payments	197
3.5.5. How much can we afford?	199
3.5.6. Problems and Exercises:	201
3.6. COMPARING MORTGAGES WITH DISCRETE AND CONTINUOUS COMPOUNDING.....	205
3.6.1. Problems and Exercises:	209
CHAPTER 4.....	211
SOLVING NUMERICALLY MORTGAGE AND ANNUITIES EQUATIONS AS PARTICULAR CASES OF IRR EQUATION	211
4.1. NEWTON-RAPHSON'S METHOD.....	213
4.1.1. Mathematical presentation.....	213
4.1.2. Problems and Exercises:	220
4.2. NUMBER OF SOLUTIONS OF IRR EQUATION.....	220
4.2.1. A numerical example of IRR equation with three roots	221
4.2.2. Problems and Exercises:	223
4.3. METHODS FOR SOLVING IRR EQUATION.....	224
4.3.1. Linear approximation of IRR equation	224
4.3.2. Quadratic approximations of the IRR equation	226
4.3.3. Quadratic solution at point zero.....	228
4.3.4. Generalization of iterative algorithms for IRR equation.....	232
4.3.5. Iteration accuracy.....	234

4.3.6. Problems and Exercises:	236
4.4. SERIES REVERSION METHOD	238
4.4.1. Computing series reversion coefficients	239
4.4.2. Series reversion equation for mortgages and annuities. Positive powers	244
4.4.3. SERIES REVERSION METHOD FOR MORTGAGES AND ANNUITIES. NEGATIVE POWERS.....	246
4.4.3.1. Mathematical algorithms	246
4.4.3.2. Series reversion algorithms' accuracy.....	249
4.4.3.3. Computational performance of series reversion algorithms	250
4.4.4. Discussion of series reversion method	251
4.4.4. Problems and Exercises:	252
4.5. APPLICATION OF METHODS FOR SOLVING IRR EQUATION IN THE BUSINESS ENVIRONMENT	252
4.5.1. Choosing the first approximate value of interest rate for iterative procedures	253
4.5.2. Factors to be considered when adopting.....	254
computational methods.....	254
4.5.3. Problems and Exercises:	255
CHAPTER 5.....	257
COMPUTATIONAL EFFICIENCY OF ALGORITHMS FOR SOLVING IRR EQUATION	257
5.1. SOFTWARE IMPLEMENTATION DETAILS	257
5.2. ACCURACY OF METHODS FOR COMPUTING INTEREST RATE.....	259
5.2.1. Comparison and hierarchy of computational methods from the accuracy perspective	259
5.3. SOLUTION ACCURACY. LIMITATIONS OF METHODS WITH REGARD TO BUSINESS SCENARIOS.....	263
5.3.1. Problems and Exercises:	271
5.4. COMPARING COMPUTATIONAL PERFORMANCE OF DIFFERENT METHODS	273
5.4.1. Problems and Exercises:	276
6. HIGH PERFORMANCE COMPUTATIONAL METHODS FOR MORTGAGES AND ANNUITIES. COMPUTING ALGORITHMS FOR FINANCIAL CALCULATORS	278
6.1. IRR EQUATION FOR MORTGAGES AND ANNUITIES	279
6.2. SPECIFIC PROPERTIES OF THE MORTGAGE IRR FUNCTION AND ITS CHARACTERISTIC POINTS.....	282
6.2.1. General form of IRR function. Solution of IRR equation.....	282
6.2.2. Asymptotic behavior of mortgage IRR function. Characteristic points	284

6.2.3. Transforming mortgage IRR equation	287
6.3. APPROXIMATING SOLUTION OF THE IRR EQUATION	289
6.4. AN ACCURACY OF X AND Y APPROXIMATIONS, AND THEIR COMPUTATIONAL PERFORMANCE	292
6.5. GENERALIZATION OF X- AND Y- APPROXIMATIONS. A-APPROXIMATION METHOD	295
6.6. ADDING CP-APPROXIMATION	298
6.7. PRACTICAL IMPLEMENTATION OF A-APPROXIMATION METHOD. ACCURACY EVALUATION. NUMERICAL EXAMPLES ...	299
6.7.1. Threshold function	300
6.7.2. Power function	302
6.7.3. Iterative algorithms	302
6.8. DEFINING AN ITERATION ACCURACY	306
6.9. PROBLEMS AND EXERCISES:.....	307
6.10. CONCLUSION	309
REFERENCES	310
INDEX.....	314