Solving the Puzzle of IRR Equation

Choosing the Right Solution to Measure Investment Success

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Preface

Choosing the right solution of IRR equation, which is the foundation of many financial methods, was always a problem. In fact, this problem impeded the wide acceptance of the IRR equation in financial industry, despite its better objectivity compared to other methods. We study the problem of multiple roots of IRR equation and introduce a novel "largest root" rule for selecting the correct solution, which effectively resolves the puzzle of IRR equation.

Many practical financial methods are based on calculation of rates of return, such as portfolio analysis and composition, risk measurement, and performance attribution. Examples of interesting and thought provoking approaches and commentaries can be found in the articles of Waring (2006), Kozhemiakin (2006), Hood (2005), Kritzman (2006), Busse (2010), which emphasize the importance of objective valuation of rates of return as a foundation of risk measurement, attribution analysis, and investment performance measurement. Calculation of much accounting data requires usage of rates of return or interest rates, such as the cost of debt and its optimal structure considered in Binsbergen (2010). The rate of return is indirectly linked to many other financial instruments. For instance, the rate of return is included in risk measurement values which relate to a company's debt capacity, in Rampini (2010). A good review of historical developments and presently used methods for computing rates of return of investment portfolios and quantitative parameters based on them is given in Spaulding (2005, 2009). There is an interesting research in Osborne (2010) that studies the relationships of multiple roots of IRR equation with the NPV method.

The financial industry uses different mathematical methods for evaluating rates of returns on investments. It was rigorously proved in Shestopaloff (2007, 2009) that all existing methods for calculating rates of return can be derived as certain approximations to the internal rate of return (IRR), so that mathematically IRR is the base method. However, besides the mathematical aspects of correct calculation of rates of return, there are other important issues related to the business side. Two of them are of paramount practical importance. First, what is the business meaning of rates of return that are obtained by different methods, and second, what is the business value of these numbers in practical applications. In other words, how should financial analysts interpret different rates of return from a business perspective? The next issue is unambiguous calculation of rates of return. For instance, it is well known that the IRR equation, in general, can have multiple roots. How do we decide which root is the right one?

This book addresses these issues. We offer solutions that have a well defined business meaning, and that can be beneficially applied toward a very wide range of problems. First, we consider the notion of *conceptual context* of rate of return. This notion has been introduced and elaborated in Shestopaloff (2008, 2009). In this book, we consider how this useful notion helps to choose the methods that are most adequate for the purpose of investment analysis, and how it helps to interpret the results of mathematical computations. In the second part, we introduce methods for resolving the problem of ambiguity of rate of return when a method produces multiple solutions. In particular, we consider how to resolve the ambiguity of solutions of the IRR equation.

The possible and, in many instances, actual ambiguity of solutions of IRR equation impedes the practical application of the IRR method, which is used not only for computing rates of return, but is intensely exploited in many other financial applications, such as calculation of implied volatility, computing yield to maturity, calculation of interest rates for various types of mortgages and annuities, etc. We introduce practical and efficient methods that allow unambiguously finding the correct, from a business perspective, solution of the IRR equation. These results are also applicable to NPV and MIRR methods, which are mathematical derivatives of the IRR equation.

We begin with the notion of investment context and its application to the evaluation of the rate of return. Then, on the basis of this notion, we classify and, in some instances, generalize existing methods for computing rates of return. In particular, we introduce the generalized IRR equation that covers all previously considered investment contexts, such as continuous and discrete compounding, non-compounding and even scenarios when no interest is paid. We also consider the properties of geometric linking algorithm and related methods, and introduce and prove the Geometric Linking Theorem that defines the exact domain of applicability of geometric linking. The set of linking algorithms, which we introduce further, does not have drawbacks of the geometric linking algorithms and adds many advantageous features. In particular, these algorithms allow linking not only period rates of return, but also linking the rates of return of different assets within the same period.

The second part of the book studies the old problem of selecting the unique correct solution of the IRR equation, when this equation has multiple roots. We introduce several robust criteria for selecting the correct solution that are based both on *mathematical* and *business* considerations, which allows us to formulate "The Rule of the Largest Root" for selecting the correct solution.